$$u_t = 75.40 - 37.70 - 25.13 - 12.57 - 8.38 - 6.28 - 5.03 - 3.97 - 3.28$$

The last but one value 3.97 of the series is assumed as gear ratio of direct drive (u_g=1). Then u_0 = 3.97, and V_{direct}=95 km/hr.

For the 9-stage gearbox transmission ratio series is of the following form:

$$U_g = 19.00 - 9.50 - 6.333 - 3.167 - 2.111 - 1.583 - 1.267 - 1.0 - 0.826$$

Instead of that, five-stage gearbox with divider may be considered.

So, the computations for mechanical gearbox with progressive series density (hyperbolic series) which is mostly close to infinitely variable transmission are significantly simplified. The advantages of the series described are the highest maximum speed (based on maximum engine power), high acceleration rate, high fuel economy, limited number of stages, low mass and design simplicity. The highest accelerating gear is usually provided for reducing fuel consumption in actual vehicle operation.

Let us discuss another case of selecting u_0 for the same train at the given maximum engine power P_{max} = 132 kW. The initial data for computation are obtained on the Central Proving Ground speed track of the primary hilly type (i_{max} up to 4%, $\psi \approx 0.03$).

Table 2

U_0	6.5	7.0	7.5	8.0	Initial values assumed
$\overline{V}$	54	56	57	58	
$\overline{Q}_D$	44	46	47	51	
k_t	1.96	1.89	1.85	1.82	
η_v	0.303	0.290	0.283	0.261	

The peak efficiency value 0.303 is obtained at U_0 = 6.5, hence the latter should be take[n] as the basis for updating drive shaft at the given engine power.

5 Conclusions

The regression analysis confirms that vehicle productiveness and fuel economy are th[e] determining factors in forming transportation cost, have linear functional relationshi[p] with it and diversely affect it. Being ranged by the rate of this influence, the first is fu[el] consumption, the second is productiveness, and the third is maintenance.

By use of integrated estimation of vehicle dynamics and economy, the diversity o[f] these properties may be eliminated, and the selection of the significant design paramete[rs] diversely affecting (engine power, maximum speed, load capacity, transmission ratio[s] etc.) may be optimized.

The rules of influence of such parameters (commonly, oppositely directional) o[n] vehicle average speed and fuel consumption are defined, with essential discrepancy [of] optimum regions; the method of eliminating this contradiction is found.

The simple and effective procedure of selecting transmission ratios of progressi[ve] ... The advantages of the method described

Technical Note: A method for quantifying front wheel bicycle hub stiffness

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1 Introduction

The use of suspension forks has become increasingly common in off-road riding. This is due to their ability to isolate the rider and bicycle from vibration, which in turn can improve handling and comfort. Suspension forks provide vibration isolation by allowing the front wheel to move relative to the rest of the bike. Thus, the bike and rider travel a smoother path than the front wheel which follows the contours of the ground.

Most suspension forks allow relative motion between the frame and front wheel by having the fork blades function as telescoping members, either expanding or contracting in length to keep the wheel on the ground. When the fork blades are extended the same amount (no relative displacement), then the hub will sit level, and the plane of the wheel will be parallel to the fork blades. If, however, one fork blade is extended more than the other one (a relative displacement), then the hub will sit at an angle. This will force the rim off-centre by an amount proportional to the relative displacement and, if large enough, into one of the brake pads. This independent motion of the fork blades is one of the major problems currently facing suspension forks.

In most suspension fork designs, there are two structural members which contribute to the overall stiffness between the fork blades: the brake bridge and the front hub. Increasing the stiffness of either of these will increase the overall stiffness, reducing the amount of relative displacement. Since brake bridges were evaluated in a recent issue of a leading consumer magazine (Anon, 1993), this project focused on the front hub.

A hub consists of several individual parts: the axle, the quick release skewer, the hub shell, and the bearings; all of which contribute to the hub's overall stiffness. However, the amount each part contributes to the overall stiffness is not well known. This fact is exemplified by the myriad of hub designs currently available.

With this in mind, this project was undertaken to evaluate the stiffness of several currently available suspension hubs and to use the stiffness data to gain a better understanding about how hub design relates to hub stiffness. This information can help hub manufacturers to produce stiffer hubs.

2 Methods

In designing the test and test fixture, the aim was to accurately reproduce the types o loads that would occur during typical off-road cycling. Two different loading conditio which would cause a relative displacement between the fork blades were identified; cas #1: the spring rates (or air pressures) in each fork leg are equal and a bump is hit at a angle which produces a side load on the wheel. In case #2: the spring rates in each fo leg are slightly different which causes a relative displacement when a bump is hit straigh on.

With the two loading conditions identified, a test fixture which could accuratel reproduce them was designed. For case #1, a moment was applied to the hub shell via th spoke flanges, whereas in case #2 a moment was applied directly to the axle. For case # a jig was devised which clamped the steerer tube while one spoke flange rested on horizontal surface. A weight was hung from the other spoke flange via a nylon strap (se Figure 1a). The same jig was used for case #2 except that the fork blade rather than th spoke flange rested on the horizontal surface. The weight was hung from the other fo blade via a clamp (see Figure 1b).

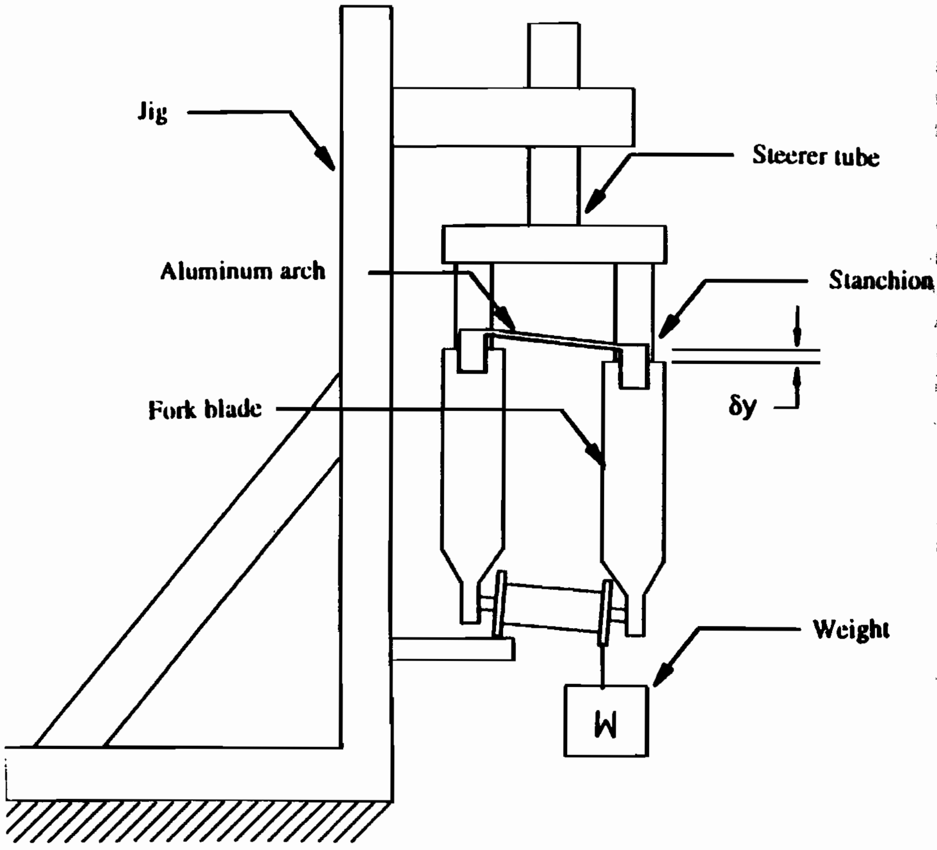

Figure 1a Loading case #1, load applied to the hug flange.

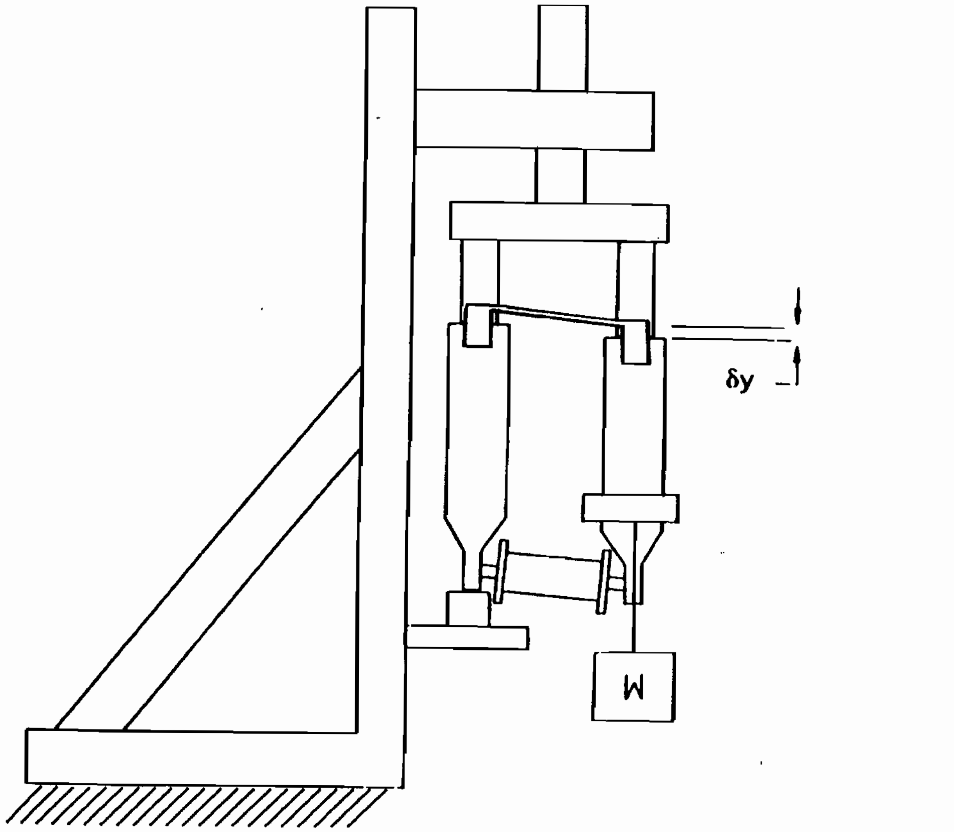

Figure 1b Loading case #2, load applied to the fork blade.

The custom jig was fabricated to hold a modified Specialized Future Shock (Specialized Bicycles, Morgan Hill, CA) such that the fork blades were vertical. Both the air and oil were removed to allow unrestricted vertical motion of the legs (the valves and seals were removed to minimize the friction). The brake arch was replaced by a thin piece of aluminium which allowed the fork blades to move relative to each other vertically while limiting the rotation of the fork blades about their stanchion tubes.

Measurements of the relative displacement of the fork blades allowed a stiffness (K) to be calculated as the moment (M) divided by the amount of displacement (δy)

$$K = M / \delta y \tag{1}$$

Here the units of stiffness are in N-m/m (or in-lbs/in).

To ensure that a pure moment was applied to the hub shell for case #1 and to the axle for case #2, a static analysis was performed. Figure 2 shows a freebody diagram of the fork and the forces acting on it for case #2 (case #1 is similar). Since it was desired to have $F_2 = W$, F_1 was eliminated by covering the horizontal surface that the fork tip rested on with a thin sheet of Teflon. Having $F_1 = 0$ meant that $F_3 = 0$. Summing forces in the vertical direction:

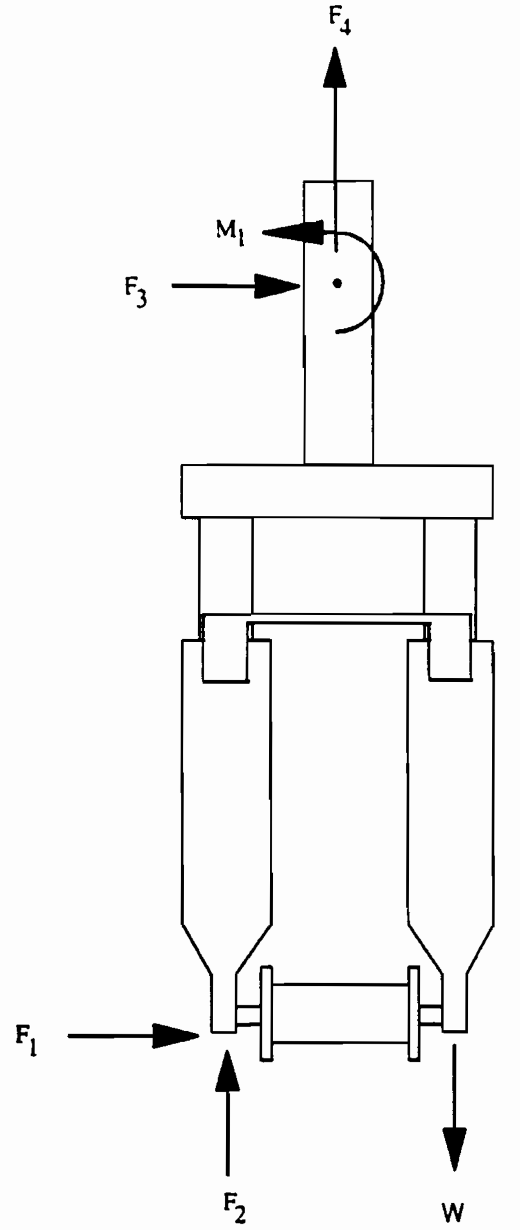

Figure 2 Freebody diagram of fork and hub for loading case #2.

But F4 = 0 because the fork blades are free to move vertically (only the bushing friction prevented vertical motion). Thus, the test fixture was capable of applying a pure moment:

$$M = M_1 = WL \quad (3)$$

where L would be the distance between the fork blades for case #2 and the distance between the spoke flanges for case #1.

Combining Equations 1 and 3 yields an expression for the stiffness as a function of the weight used, the distance L, and the measured relative displacement:

$$K = WL / \delta y \quad (4)$$

The clamping force generated by a standard quick release was determined by threading a nut onto a quick-release skewer, closing the lever, and removing the nut with a torque wrench. It was determined that 3.4 –5.7 N-m (30–50 in-lbs) of torque would be needed to imitate the normal clamping force on the hub. However, the use of 3.4 N-m caused the threads on the quick release to strip during preliminary testing. The preliminary test also demonstrated that the torque did not affect the relative stiffness ranking within the 2.3 to 5.7 N-m torque range. That is, if hub A was stiffer than hub B at one torque value, then it was also stiffer at another torque value.

The weight (W) used in the tests was determined experimentally. A relative motion between the fork blades of 1 mm (0.04 in) will cause the rim to move about 3 mm (0.12 in), enough for the rim to rub the brake pad. For the Shimano XT hub, a weight of 200 N (45 lbs) caused a relative displacement of 0.9 mm (for case #2). Based on this, 200 N was used as the weight for all tests.

The testing protocol for each hub was as follows: the hub was clamped into the fork, a dial indicator was zeroed, the weight was applied to the spoke flange (case #1) and the relative displacement of the fork blades was recorded from the dial indicator that was mounted on one of the fork blades (not shown in Figure 1). The same hub was tested five times in a row. Then the tests were repeated with a different hub until each hub had been tested. The hubs were then randomized and tested again, cycling through all the hubs twice (i.e. each hub was tested 10 times). The entire process was then repeated for loading case #2. Knowing the dimension of each hub, the stiffness was finally calculated using Equation 4.

Several hubs were capable of being bolted on (rather than using the quick release). The stiffness of these hubs was also determined using the above protocol except that for the tests the bolts were tightened to a 'reasonable amount'. This was done because each bolt had different threading and, thus, equal torques would not result in equal clamping forces. A 'reasonable amount' was defined as the torque that could be applied using tools that would typically be carried during a ride.

3 Results

The results for both loading cases are shown in Table 1, with the hubs ranked by their mean stiffness as determined by loading case #1. Both the mean and a 95% confidence interval for the stiffness values are presented. Table 1 also gives a mass of the hub, and dimensions of the axles (dimensions are referenced to Figure 3).

For discussion purposes, the stiffnesses of several axles were determined and are listed in Table 2. Again, the mean and 95% confidence intervals are presented. In comparing the results of Tables 1 and 2, it is evident that the axle alone accounts for most of the hub stiffness. In fact, two out of the four axles tested were statistically indistinguishable from the corresponding hubs.

able 1 Hub stiffness results for testing using the skewer.

UB	Stiffness (kN-m/m) Case #1 (flanges) Mean	Case #1 95% confidence	Case #2 (fork blades) Mean	Case #2 95% confidence	Mass (g)	Dimensions (mm) a	b	c	ID	Face
llseye	36.36	35.33 – 37.39	35.37	34.29 – 36.46	184.5	8.7	12.0	12.0∇	5.7	25.4
ugi	33.32	32.58 – 34.06	33.28	32.00– 34.55	184.3	8.8	12.0	16.0	6.4	21.1
ershey (B)	32.99	32.22 – 33.76	32.92	32.39 – 33.45	160.7	9.0	18.7	19.0	8.1	21.8
hite Industries	32.50	31.42 – 33.58	33.97	33.01 – 34.93	152.8	8.8	17.0	17.0	N/A*	20.1
riel	31.84	30.25 – 33.44	34.62	33.74 – 35.51	141.0	9.0	12.0	15.9	5.1	21.5
ertical Descent	30.65	29.53 – 31.77	29.18	28.83 – 29.54	140.9	9.0	12.0	12.7	5.4	21.4
NT (B)	30.60	30.12 – 31.08	31.68	31.38 – 31.99	132.6	8.9	20.0	25.4	N/A*	19.9
ingle	30.12	29.46 – 30.79	30.81	30.11 – 31.51	157.8	9.0	12.0	16.0	5.3	18.9
imano XT(B)	30.02	29.39 – 30.64	30.48	30.15 – 30.81	151.9	8.8	8.9/12.0#	10.3	5.5	19.4#
achine Tech	29.97	29.31 – 30.63	31.20	30.44 – 31.96	154.4	9.0	12.0	16.0	5.6	21.8
imano LX	29.21	27.73 – 30.70	28.52	26.93 – 30.12	160.6	8.5	9.2	9.2	5.5	19.4
ope	29.18	28.28 – 30.07	29.39	28.25 – 30.54	164.3	8.9	11.9	16.0	5.2	22.5
merican Classic	28.23	27.63 – 28.83	28.67	28.27 – 29.08	147.3	8.6	11.8	15.8	5.2	19.2
ontour	25.56	24.58 – 26.54	27.18	25.89 – 28.48	126.5	8.8	12.0	12.0	5.2	21.2
avic	25.49	24.04 – 26.94	24.42	23.65 – 25.19	159.7	8.8	14.7	N/A@	5.2	17.7
imano XT(A) monst. flange locknuts	25.24	22.18 – 28.31	28.73	27.63 – 29.83	138.8	8.8	8.8	8.8	5.4	22.1
ukeproof (B)	24.25	23.70 – 24.81	23.45	23.10 – 23.80	144.5	8.7	12.5	17†	5.2	21.8
Iterrain	23.43	22.05 – 24.82	25.46	24.22 – 26.71	147.0	8.9	8.9	8.9	5.3	19.0
NT (A)	22.05	21.12 – 22.97	22.92	22.39 – 23.45	138.2	8.9	16.9	25.4	N/A*	16.9
imano XT (A)	21.60	21.13 – 22.08	23.95	23.25 – 24.66	136.8	8.8	8.8	8.8	5.4	19.0
ershey (A)	21.49	21.03 – 21.96	22.02	21.42 – 22.62	126.0	9.0	16.8	19.0	N/A*	16.8
ukeproof (A)	21.05	20.44 – 21.65	21.44	20.60 – 22.27	113.6	8.9	9.3	17†	5.6	17.1

otes: All masses are for hub alone; Advent skewer adds an additional 30 g; ∇ Bullseye utilizes a bearing spacer (17 mm O.D., 12.5 mm I.D.) that acts as axle sleeve
White Industries, TNT (A&B), and Hershey (A) hubs utilize oversized hollow axles that do not conform to Figure 3. † Nukeproof (A & B) hubs use 17 mm fluted axle
- Shimano XT (B) uses 3 piece axle that does not conform to Figure 3 and a puzzle-piece face that fits in the drop-out @ Dimension not available

3 Stiffness results for bolt-on and oversized skewer hubs. Hubs are bolt-on unless otherwise specified.

HUB	Stiffness (kN-m/m) Case #1 (flanges) Mean	Case #1 95% confidence	Case #2 (fork blades) Mean	Case #2 95% confidence	Mass (g)
e	40.51	37.40 – 43.62	45.21	43.18 – 47.23	212.6
)	34.26	32.48 – 36.04	36.50	33.67 – 39.32	152.1
	33.66	31.81 – 35.50	36.98	34.78 – 39.18	255.0
	33.31	30.90 – 35.71	35.89	34.82 – 36.95	165.4
(A)	32.35	31.11 – 33.59	33.62	32.19 – 35.05	155.6
	29.48	28.08 – 30.88	29.67	27.07 – 32.26	229.3
)	27.97	36.67 – 29.27	29.84	28.33 – 31.36	157.7

All masses include bolts or skewer

* - Hugi hub uses a 6 mm skewer

† - Pulstar hub uses a 9 mm skewer

Table 2 Stiffness of selected axles alone. Loading case #2 only.

Axle	Stiffness (kN-m/m) Case #2 (fork blades)	
	Mean	95% confidence
White Industries	29.98	29.18 – 30.79
Shimano XT (A) w/Monster flange locknuts	29.80	28.79 – 30.80
Shimano XT (A)	24.15	23.91 – 24.40
TNT (A)	20.73	19.56 – 21.89

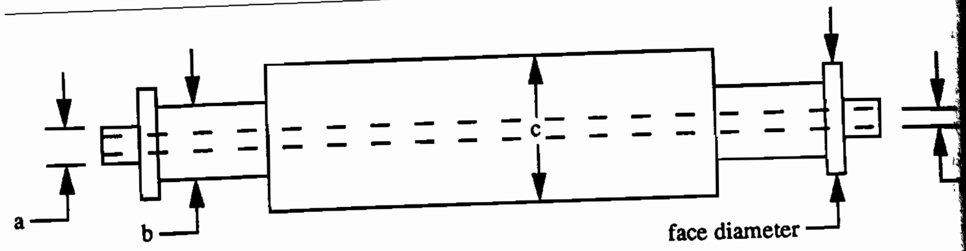

Figure 3 Nomenclature for axle dimensions.

As mentioned earlier, several hubs allowed the use of bolts rather than a quick rele skewer. Table 3 lists the stiffnesses for the bolt-on hubs tested. For these tests, the use different types of bolts caused them to have different clamping forces. The stiffnes given for the bolt on hubs are valid but should not be directly compared to each oth Rather, these numbers are provided to give an idea of a hub's performance when bol on and to provide insight as to how clamping force affects stiffness.

4 Discussion

From the results, several inferences can be made concerning the effects that the hub sh axle, clamping force, and face diameter have on the overall stiffness. Each of the f tests that compared the stiffness of an assembled hub with the stiffness of its axle al demonstrated that most of the hub's stiffness comes from the axle. Additiona comparing the test results of the Ringle and Machine Tech hubs, which have very sim designs except for the fact that the Machine Tech hub shell is much larger, supports idea that the hub shell contributes little to the overall stiffness.

To get a feel for how axle design can affect the stiffness of a hub, it is usefu calculate the deflection of a double cantilever beam:

$$\delta y = \frac{32ML^2}{3\pi\left(D_o^4 - D_i^4\right)E}$$

where M is the magnitude of the applied moment, D_o is outside diameter, D_i is the in diameter, E is the elastic modulus, and L is the length. Since the axle length

Equation 5 suggest two ways to increase stiffness. The amount of relative displacement of the fork tips can be decreased by using a material with a larger elastic modulus (E). Steel has an elastic modulus which is nearly three times as much as that of aluminium. For two axles of similar dimensions, a steel one should theoretically have a third of the relative displacement. Unfortunately, for the sample of hubs tested there were no steel and aluminium axles with the same dimensions so there was no chance for comparison.

Steel is also nearly three times as dense as aluminium, so most manufacturers use aluminium axles. The notable exception was the top performing White Industries hub, which uses a hollow steel axle.

Increasing the moment of inertia term, $\left(D_0^4 - D_i^4\right)$, is another way to decrease the relative displacement. In this term, the diameters are raised to the fourth power, amplifying the effect of increasing their values. If the inside diameter is held constant while the outside diameter is increased, then the stiffness of the axle will increase dramatically. For a standard axle with a 5 mm inside diameter, increasing the outside diameter from 9 to 12 mm will cause the moment of inertia to be nearly 2.5 times larger, with a theoretical decrease in displacement of 60%. The actual decrease in displacement will be slightly smaller due to the compliance between the axle and fork (discussed below).

The effect of axle design can be seen experimentally by comparing the result of the Suntour and Machine Tech hubs which both use 7075 aluminium axles with similar face diameters. The main difference between the hubs (beside the hub shell) is that the Machine Tech axle increases to 16 mm between the bearings whereas the Suntour axle is a constant diameter of 12 mm. From Table 1, it is obvious that the Machine Tech hub is significantly stiffer than the Suntour hub.

Not only do the results indicate that a larger axle results in a stiffer hub, but they also demonstrate that the axle is one of the main contributors to the overall stiffness of the hub. The Ringle, Machine Tech, and Hope hubs all utilize nearly identical axle designs and use the same bearings, but use different shell designs. Not surprisingly, all three hubs had nearly identical stiffnesses.

The stiffness of the axle, however, is not the only contributor to the overall stiffness of the hub. Since the axle functions as a link between the two fork blades, the compliance of the hub/fork interface is also important. To gain insight as to how much compliance there is between the axle and the fork blade, it is worth computing the relative displacement due to the axle alone from Equations 5. Using $D_o = 9$ mm, $D_i = 5$ mm, $L = 100$ mm (the dimensions of the Shimano XT axle), $E = 200$ GPa, and $M = 20$ N-m yields a relative displacement of 0.57 mm (0.022 in), whereas the actual axle allowed 0.9 mm (0.036 in) in displacement. Thus, the lack of a perfect connection between the fork and hub accounts for almost half of the deflection for this particular case.

Obviously, the connection between the fork and the axle is an important one. The two factors that affect the compliance between the hub and fork are the clamping force and face diameter. The clamping force is generated by the axle bolts or skewer and keeps the axle faces and the fork dropouts in contact. The notion that a larger clamping force will decrease compliance is supported by the fact that all of the hubs capable of being bolted on performed better when they were bolted into the dropouts (Table 3) than when they were held by the skewer (Table 1). However, the difficulty of

The other factor that limits the amount of compliance between the axle and fork is th diameter of the axle faces. The larger the axle face, the lower the compliance. As the hu rotates about the contact point between the dropout and the face, the bolt or skew stretches. For a given amount of bolt elongation, a larger face will result in less ax rotation (see Figure 4). This was demonstrated by installing oversized Monster Flan locknuts on the Shimano XT hub. Face diameter increased from 19 to 22 mm whi resulted in the mean stiffness increasing significantly from 21.6 to 25.2 kN-m/m (ca #1). A similar result is observed between the TNT (A) and TNT (B) hubs (face diamete of 16.9 to 19.9 mm respectively) which increased the mean stiffness significantly fro 22.0 to 30.6 kN-m/m (case #1).

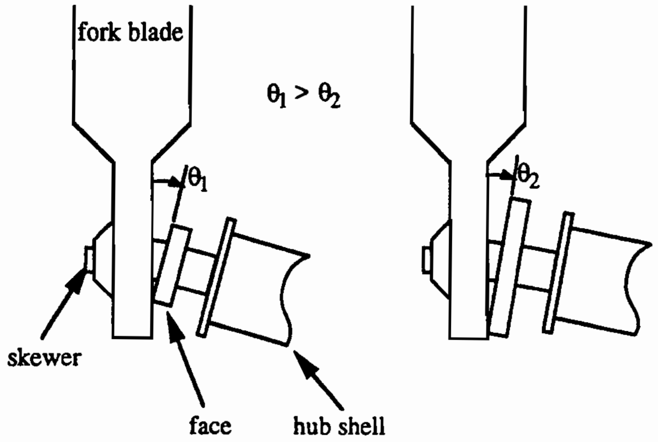

Figure 4 For a given amount of skewer elongation (exaggerated), a large face diameter will res in less axle rotation (θ).

Finally, in interpreting the results of this paper, it must be remembered that there a many factors that affect the stiffness of an assembled wheel, not merely the stiffness the wheel's hub. This study only investigated the stiffness given to a suspension fork by hub and did not consider other factors or effects.

5 Conclusions

The hubs in this test have many different axle shapes and sizes, and represent seve approaches to increasing a hub's stiffness. The results demonstrated several points th should be taken into account in designing suspension hubs:

- The axle provides most of the stiffness of the hub with the hub shell contributing little.
- The stiffest axles were typically oversized (16 mm) aluminium axles.
- The compliance between the axle and fork dropout is also a major factor in the overall stiffness.
- Increased clamping forces significantly increase stiffness.

Acknowledgements

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Reference

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